Kelly's Criterion for Option Investment

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Outline:

- Review Kelly's Problem
- Application to Option Investing
- Option Investing with Catastrophic Loss
- Joint Investments, no Correlation

- Joint Investments, with Correlation
- The Downside to Kelly

An aside, recommended reading, “Fortune’s Formula”, by William Poundstone.

Kelly's Problem

Kelly's Problem: we will make repeated bets on the same positive expectation gamble, how much of our bankroll should we risk each time?

From before the fraction to bet is:

$$f = \frac{\text{expectation per unit bet}}{\text{gain per unit bet}}.$$

or as some put it edge/odds.

Example

Say the gamble is to flip a fair coin, the payoff odds are 2:1. How much should we bet?

The expected or average payoff per play is

$$(\$2)\frac{1}{2} - (\$1)\frac{1}{2} = \$0.50.$$

Upon a win, for a $1 bet we get our bet back and $2 besides, so the gain per unit bet is 2; hence the Kelly fraction is

$$f = \frac{1/2}{2} = .25$$

or 1/4 our bankroll.

Virtues of Kelly

- our investment is compounded so we get exponential growth (or decay)
- our growth of capital is maximized
- we can not be wiped out – e.g. for an even money 60/40 investment, $f = .2$, upon a loss we still have 80% of our bankroll.

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  Starting with $20,000, after 11 losses in a row we still have $1,718, the probability of 11 losses in a row is $1/25{,}000$.

Later we discuss the downside of the Kelly fraction.

Kelly applied to Option Investing

While stock investments are more free-form, many option investments have common ground with gambles:

- fixed terms
- a definite time horizon
- a payoff settlement at expiration

Hence with the proper statistics, we can use the Kelly criterion to determine optimal investment levels while protecting against a string of reverses.

Gathering Statistics

We need to identify the characteristic features of a specific option play we customarily make. Then record the particulars and results of that play over many implementations.

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While thinking about this, I got a perfect gift in the form of the club's February presentation by Steve Lenz and Steve Papale of OptionVue.

OptionVue Credit Spread Data

12/05/07 Performance Analysis for Account: AIB Credit Spread DM 2

Options

Account: AIB Credit Spread DM 2 | Close

Gain/Loss | Performance Graph | Performance Analysis | Performance Summary | List of Accts | Dividends | Interest, Transfe

Period from: Jan 1, 2001 Thru: Nov 30, 2007 Asset Symbol (blank=all):

Net Profit	**27,310.50**
Total Gains	43,502.00
Total Losses	-16,191.50
Ratio gains / losses	2.69
Period of time (days)	2524
Total Number of trades	75

Orig. investment	15,000.00	01/02/01
Net D/W after start	0.00	
Ending value	42,310.50	
Lowest value	15,000.00	
Highest value	44,573.00	10/30/07
Max drawdown	2,610.50	04/30/03 - 06/30/03
Max % drawdown	17.4%	
Return	182.0%	
Annualized return	26.3%	

Number of winning trades	66	Number of losing trades	9
Percent winners	88%		
Largest winning trade	1,688.00	Largest losing trade	-2,837.00
Average winning trade	659.12	Average losing trade	-1,799.06
Average trade (win, loss)	364.14 = .88 (659.12) − .12 (1,799.06)		
Ratio avg win / avg loss	0.37		
Average winning trade length	34	Average losing trade length	20
Max consecutive winners	16	Max consecutive losses	2

Past results are not necessarily indicative of future performance.

OptionVue Credit Spread Continued

The data we need is: over 75 trades

- number that gained money: 66, 9 lost money
- average gain per winning trade: $659.12
- average loss per losing trade: $1,799.06

I regard the average loss per losing trade as the "bet size".

Credit Spread Risk Fraction

We calculate the following needed for "edge over odds"

win prob. $\frac{66}{75} = .88,\quad$ gain per unit bet $\frac{659.12}{1799.06} = 0.366.$

Hence expectation $= (.366) * .88 - (1) * .12 = .202.$

And so the Kelly risk fraction is

$$f = \frac{.202}{.366} = .552.$$

Accounting for Catastrophic Loss

But we can do more with the data. Note that we have "maximum loss" information. This can be regarded as catastrophic loss and taken into account.

Let p be the probability of a win, q the probability of a loss, and r the probability of a catastrophic loss. Let γ be the gain per unit bet and λ the size of the catastrophic loss per unit bet.

We now rederive the Kelly fraction.

Kelly Fraction for Catastrophic Loss

The expectation is now

$$E = \gamma p - q - \lambda r.$$

The expected growth rate (from the previous talk) is

$$E(g) = p \log(1 + \gamma f) + q \log(1 - f) + r \log(1 - \lambda f)$$

And the optimal fraction is the root of the quadratic equation

$$0 = E - f\left(p\gamma(1 + \lambda) + q(\gamma - \lambda) + r\lambda(\gamma - 1)\right) + \gamma\lambda f^2$$

Credit Spreads with Catastrophic Loss

The new parameters are now

- prob. of a win $p = .88$
- prob. of an avg loss $q = \frac{8}{75} = .1067$
- prob. of a catastrophic loss $r = \frac{1}{75} = .0133$
- gain per unit bet $\gamma = \frac{659.12}{1669.32} = .395$
- catas. loss per unit bet $\lambda = \frac{2837}{1669.32} = 1.70$

OptionVue Credit Spreads with Catastrophic Loss

Solve the quadratic we get

$$f = .458$$

or about 46%.