# Kelly's Criterion for Option Investment

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#### Outline:

- Review Kelly's Problem
- Application to Option Investing
- Option Investing with Catastrophic Loss
- Joint Investments, no Correlation

- Joint Investments, with Correlation
- The Downside to Kelly

An aside, recommended reading, "Fortune's Formula", by William Poundstone.

#### Kelly's Problem

Kelly's Problem: we will make repeated bets on the same positive expectation gamble, how much of our bankroll should we risk each time?

From before the fraction to bet is:

$$f = \frac{\text{expectation per unit bet}}{\text{gain per unit bet}}$$
.

or as some put it edge/odds.

#### **Example**

Say the gamble is to flip a fair coin, the payoff odds are 2:1. How much should we bet?

The expected or average payoff per play is

$$(\$2)\frac{1}{2} - (\$1)\frac{1}{2} = \$0.50.$$

Upon a win, for a \$1 bet we get our bet back and \$2 besides, so the gain per unit bet is 2; hence the Kelly

fraction is

$$f = \frac{1/2}{2} = .25$$

or 1/4 our bankroll.

#### Virtues of Kelly

- our investment is compounded so we get exponential growth (or decay)
- our growth of capital is maximized
- we can not be wiped out e.g. for an even money 60/40 investment, f=.2, upon a loss we still have 80% of our bankroll.

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  - Starting with \$20,000, after 11 losses in a row we still have \$1,718, the probability of 11 losses in a row is 1/25,000.

Later we discuss the downside of the Kelly fraction.

# Kelly applied to Option Investing

While stock investments are more free-form, many option investments have common ground with gambles:

- fixed terms
- a definite time horizon
- a payoff settlement at expiration

Hence with the proper statistics, we can use the Kelly criterion to determine optimal investment levels while protecting against a string of reverses.

#### **Gathering Statistics**

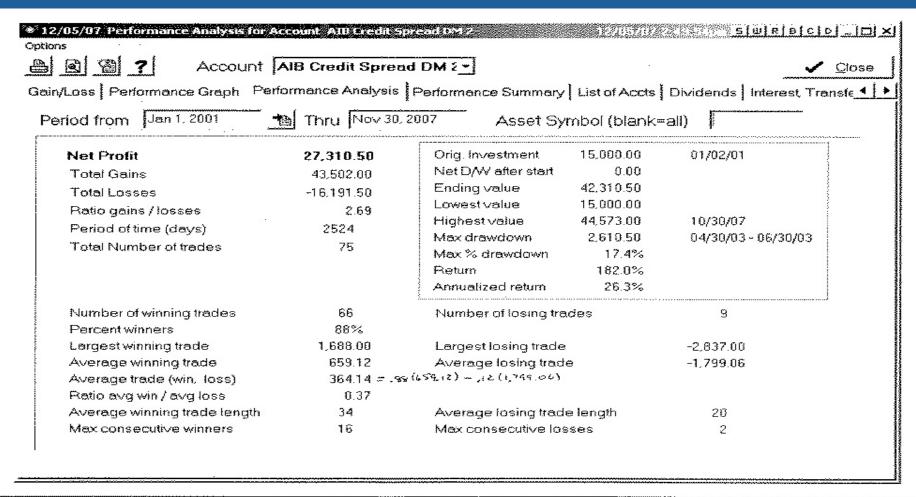
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#### **Gathering Statistics**

We need to identify the characteristic features of a specific option play we customarily make. Then record the particulars and results of that play over many implementations.

While thinking about this, I got a perfect gift in the form of the club's February presentation by Steve Lenz and Steve Papale of OptionVue.

#### OptionVue Credit Spread Data



### OptionVue Credit Spread Continued

The data we need is: over 75 trades

- number that gained money: 66, 9 lost money
- average gain per winning trade: \$659.12
- average loss per losing trade: \$1,799.06

I regard the average loss per losing trade as the "bet size".

#### **Credit Spread Risk Fraction**

We calculate the following needed for "edge over odds"

win prob. 
$$\frac{66}{75} = .88$$
, gain per unit bet  $\frac{659.12}{1799.06} = 0.366$ .

Hence expectation = (.366) \* .88 - (1) \* .12 = .202.

And so the Kelly risk fraction is

$$f = \frac{.202}{366} = .552.$$

#### **Accounting for Catastrophic Loss**

But we can do more with the data. Note that we have "maximum loss" information. This can be regarded as catastrophic loss and taken into account.

Let p be the probability of a win, q the probability of a loss, and r the probability of a catastrophic loss. Let  $\gamma$  be the gain per unit bet and  $\lambda$  the size of the catastrophic loss per unit bet.

We now rederive the Kelly fraction.

#### Kelly Fraction for Catastrophic Loss

The expectation is now

$$E = \gamma p - q - \lambda r.$$

The expected growth rate (from the previous talk) is

$$E(g) = p \log(1 + \gamma f) + q \log(1 - f) + r \log(1 - \lambda f)$$

And the optimal fraction is the root of the quadratic equation

$$0 = E - f(p\gamma(1+\lambda) + q(\gamma-\lambda) + r\lambda(\gamma-1)) + \gamma\lambda f^{2}$$

#### Credit Spreads with Catastrophic Loss

The new parameters are now

- lacksquare prob. of a win p=.88
- prob. of an avg loss  $q = \frac{8}{75} = .1067$
- prob. of a catastrophic loss  $r = \frac{1}{75} = .0133$
- gain per unit bet  $\gamma = \frac{659.12}{1669.32} = .395$
- catas. loss per unit bet  $\lambda = \frac{2837}{1669.32} = 1.70$

# OptionVue Credit Spreads with Catastrophic Loss

Solve the quadratic we get

$$f = .458$$

or about 46%.