Factors Affecting Option Prices

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- Option Strategies
- Payoff Graphs
- Pricing Factors:

Outline continued:

- Probability
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- Time to Expiration
- Volatility
- Risk-free Rate
- Natural Logarithm and the Normal Distrubution

- Psychological Effects

Option Strategies

There are dozens of option strategies, most possessing colorful names: Bear Spread, Bull Spread, Butterfly, Iron Condor, Calendar Spread, $\cdots$.

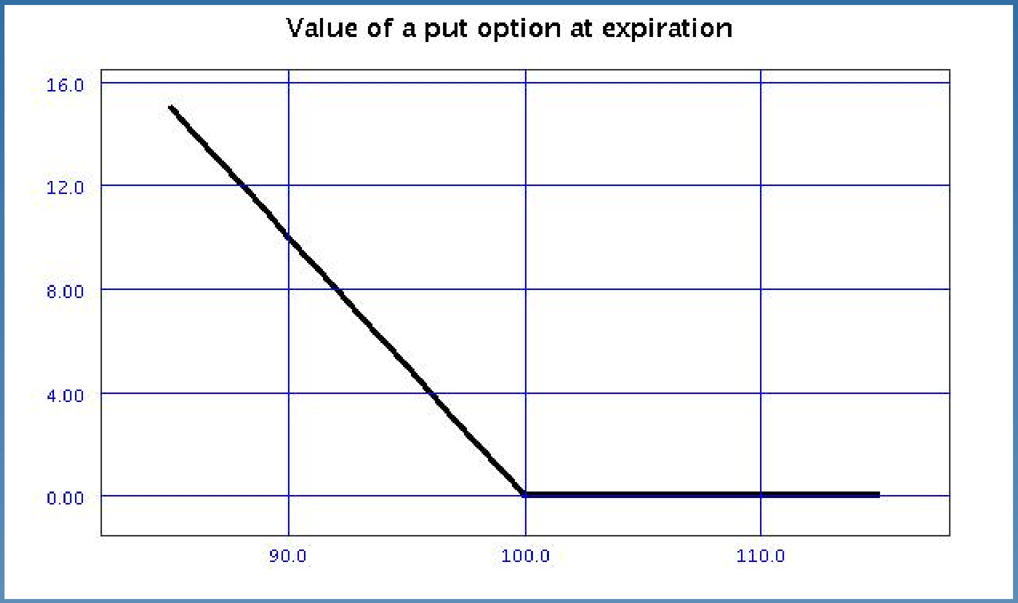
But they are all constructed from two basic options: the *put* and the *call*.

Puts and Calls

A **put** is a contract giving the *owner* the right to *sell* 100 shares of the underlying to the *maker* at the *strike price* on the *expiration date*.

If the stock's price at expiration exceeds the strike price, then the contract has zero value ("expires worthless").

Put Payoff Graph – Payoff vs Stock Price at Expiration



The option is In-The-Money (ITM) in the sloping part of the graph, (spot below strike) and Out-of-The-Money

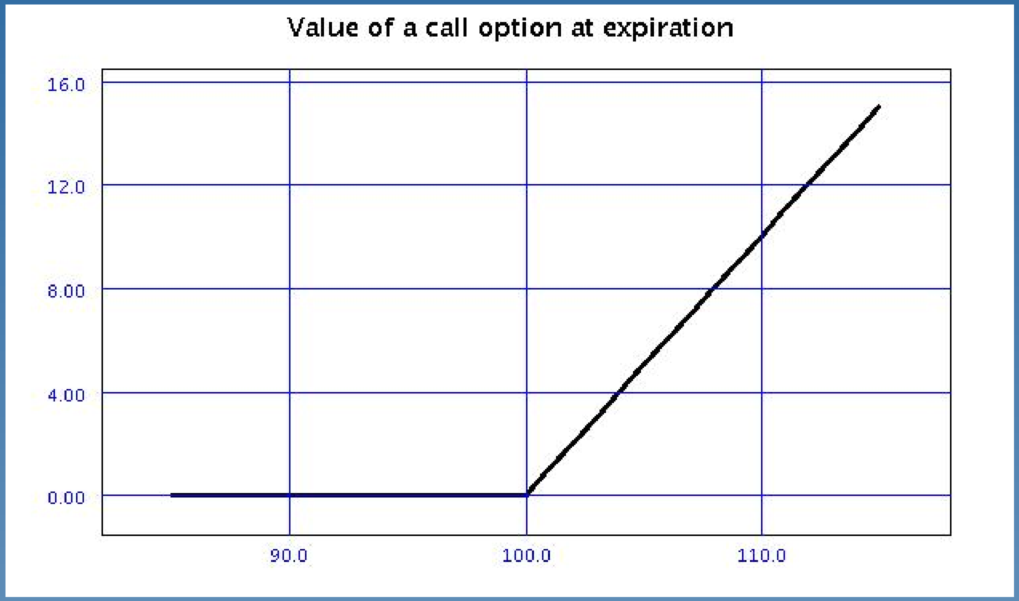
(OTM) in the flat part of the graph. Between those two, at the strike price, the option is At-The-Money (ATM).

Puts and Calls

A **call** is a contract giving the *owner* the right to *buy* 100 shares of the underlying from the *maker* at the *strike price* on the *expiration date*.

If the stock's price at expiration is less than the strike price, then the contract expires worthless.

Call Payoff Graph – Payoff vs Stock Price at Expiration



The option is In-The-Money (ITM) in the sloping part of the graph, (spot above strike) and Out-of-The-Money

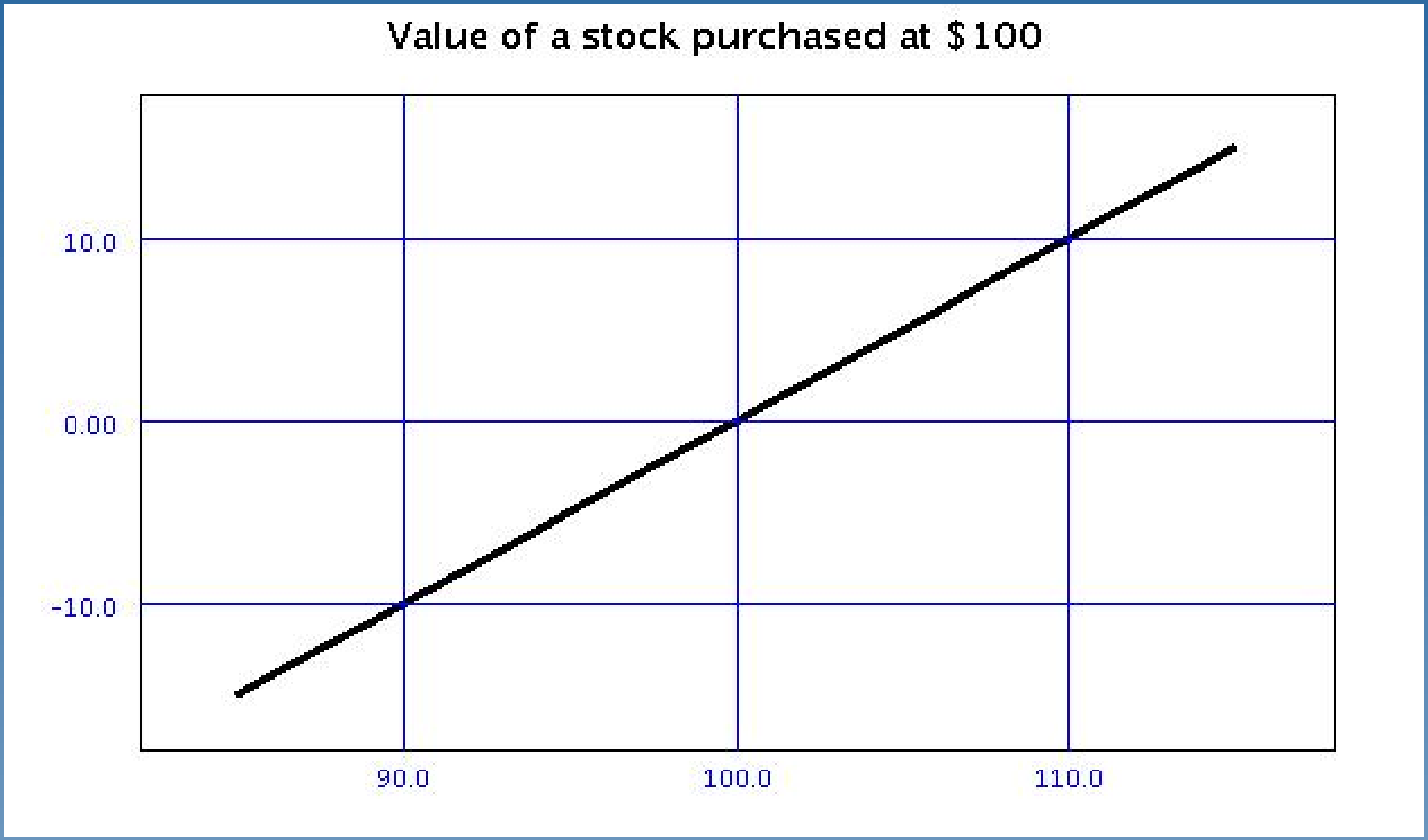
(OTM) in the flat part of the graph. At the angle between those the option is At-The-Money (ATM).

The Price of the Option

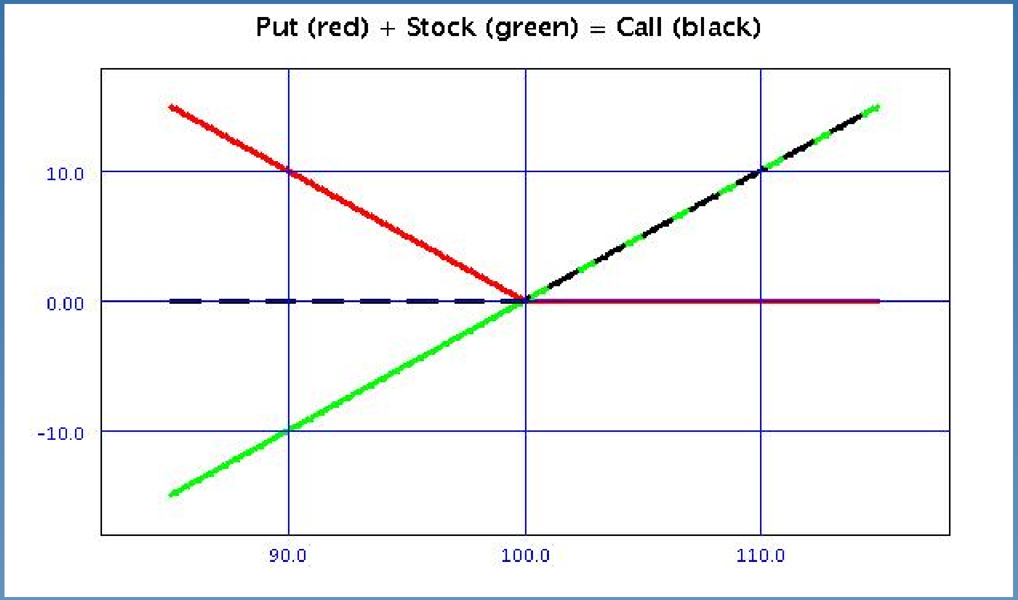
In either case, the maker receives a payment for the underwriting service called the *premium*; this is the price of the option.

It is this price that we want to understand.

Stock Ownership Payoff Graph



Put/Call Parity, $(S - K) + P = C$ (at expiration)



Thus we can *replicate* a call by buying a put, buying the underlying, and selling a bond paying the strike price.

Put/Call Parity, $P = C - (S - K)$ (at expiration)

Alternatively, we can replicate a put by buying the call, buying a bond paying K, and selling the underlying.

In this way we can assume there is only one basic option, for example, a call.

Put/Call Parity, Note

Put/call parity at any time t before expiration T is given by, (K is the strike price, non-dividend paying stock)

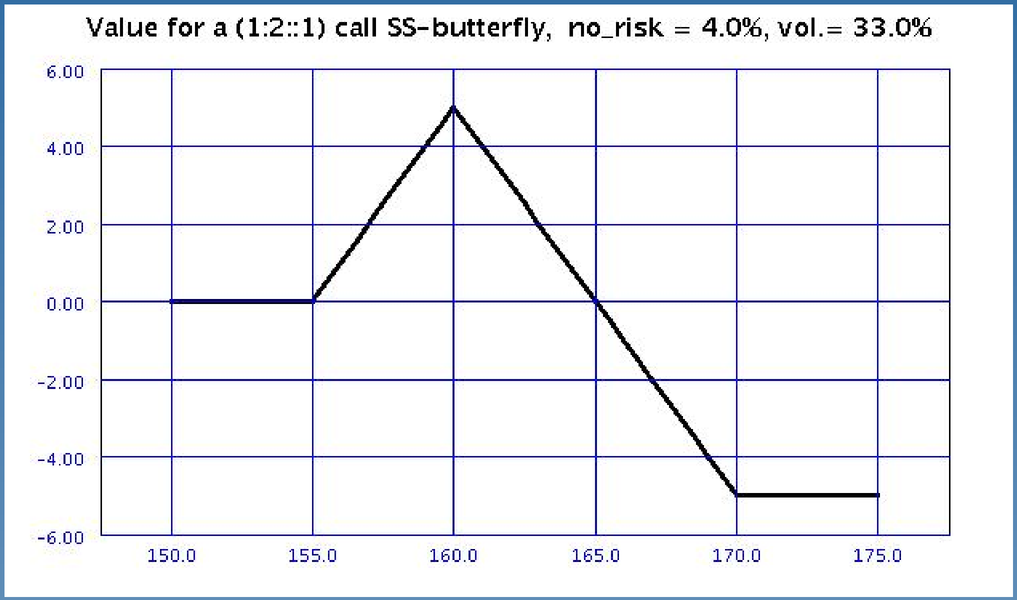
$$S_t + P_t = C_t + Ke^{-r(T-t)}.$$

(Paying for a stock today with delivery at expiration is a "forward".)

Combinations of Call and Put Options

As previously noted, more complicated trades are just combinations of calls or puts. For example a 1:2::1 skip-strike butterfly consists of buying 1 call at a low strike, selling 2 calls at the next higher strike, skip a strike, and buy another call at the 4th strike.

Standard 1:2::1 Butterfly



Constructing Payoff Graphs

For calls: work left to right, start horizontally, change direction up 1 for 1 beginning at the strike price for each long call (2 for 1 if 2 contracts, etc) and change direction down 1 for 1 for each short call (2 for 1 if 2 contracts, etc).

For puts: work right to left, start horizontally, change direction up 1 for 1 beginning at the strike price for each long put (2 for 1 if 2 contracts, etc) and similarly change direction down 1 for 1 for each short put.

Factors Affecting a Call Option – Probability

The probability that the option will expire in the money is the central factor affecting the value of an option. Although the probability itself is not directly observable, the other factors operate through it. An option value is "high" if the probability is near 100% that the underlying will finish ITM and low if the probability is near 0.

An example. Fix the stock price at $100.00, the riskfree rate at 4%, the volatility at 40% and the time to expiration at 60 days. For various strike prices from $90 to $110 we get the following.

strike	call price	ITM prob.	time value
90.00	12.81	0.729	2.81
91.00	12.10	0.706	3.10
92.00	11.41	0.682	3.41
93.00	10.75	0.658	3.75
94.00	10.10	0.633	4.10

strike	call price	ITM prob.	time value
95.00	9.49	0.609	4.49
96.00	8.89	0.584	4.89
97.00	8.33	0.559	5.33
98.00	7.79	0.533	5.79
99.00	7.27	0.509	6.27
100.00	6.77	0.484	6.77
101.00	6.31	0.459	6.31

(Why is ITM probability less than 50% at 100, when the stock price equals the strike price?)

strike	call price	ITM prob.	time value
102.00	5.86	0.435	5.86
103.00	5.44	0.412	5.44
104.00	5.04	0.389	5.04
105.00	4.67	0.366	4.67
106.00	4.32	0.345	4.32
107.00	3.98	0.324	3.98
108.00	3.67	0.303	3.67
109.00	3.38	0.284	3.38
110.00	3.11	0.265	3.11

Factors Continued – Strike Price vs Stock Price

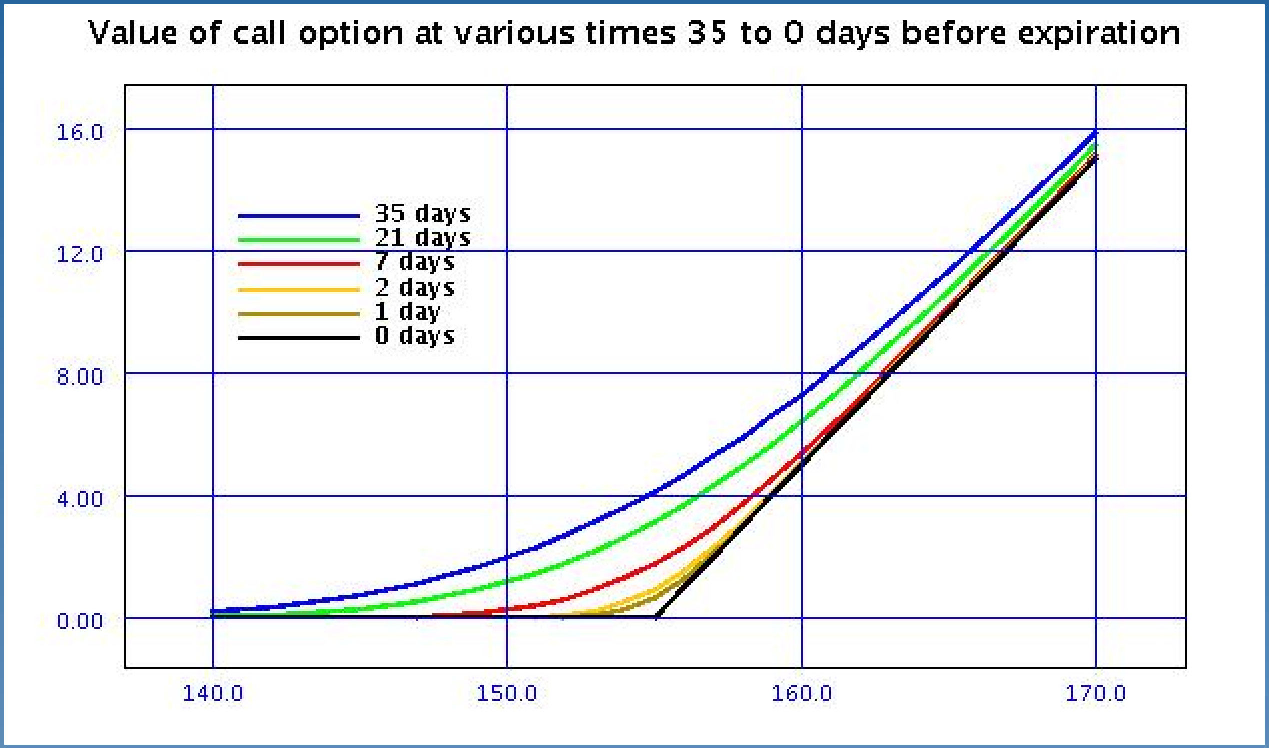
As we saw above, a call (or put) option price has two components: the *intrinsic value* and the *time value*.

The intrinsic value equals the extent to which the option is ITM. Hence the deeper in the money, the more the option costs 1 for 1.

Factors — Time to Expiration

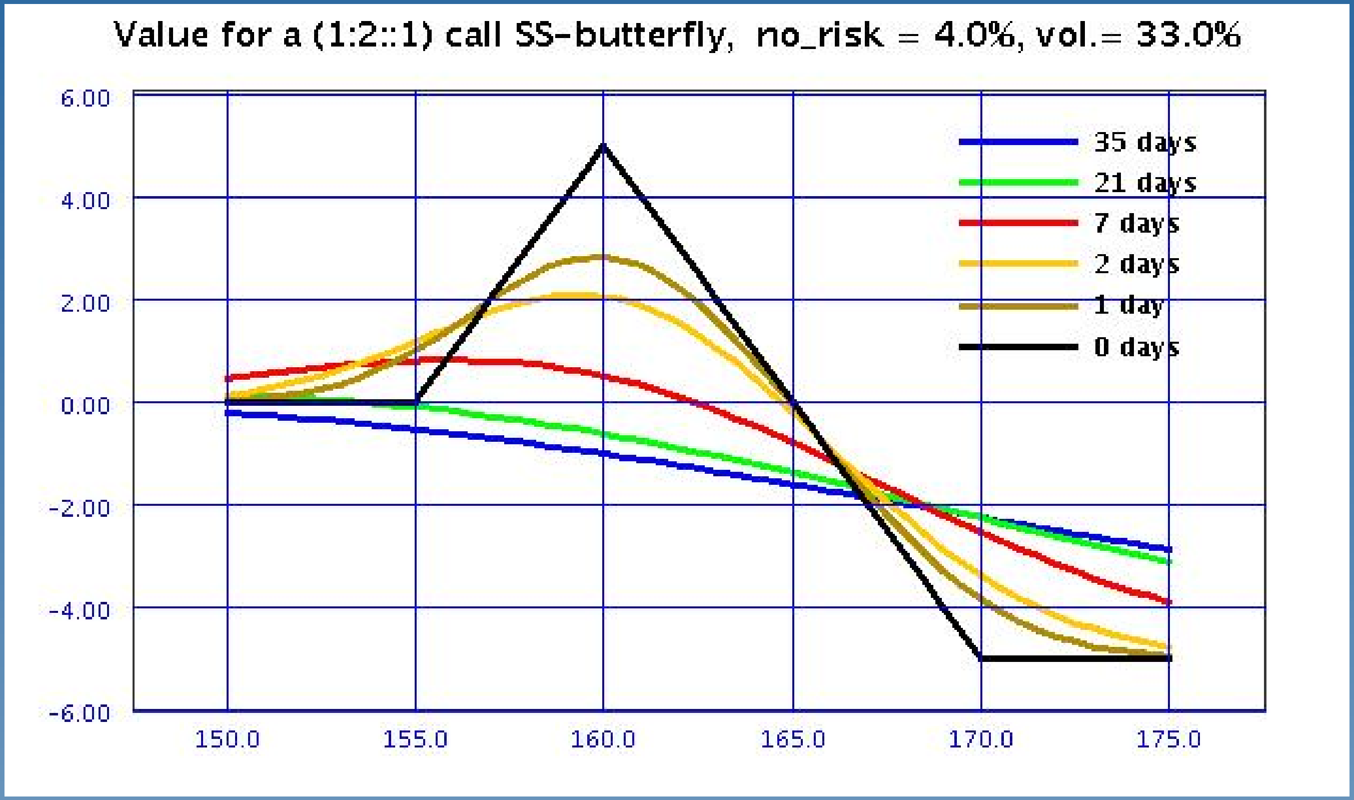
As mentioned on the previous slide, the second component of an option price is *time value*.

With lots of time remaining to expiration, the price of the underlying stock has a good chance of moving a great deal; but as expiration nears, the chance of a large price movement diminishes and becomes zero at expiration itself. As a result, the value of the option looks more and more like the payoff graph. Thus if the option is OTM, the price tends to zero, and if ITM the price tends to the intrinsic value.



Always concave up, the time value is maximum at the strike.

Time Decay Plot of a 1:2::1 Butterfly



Taking Advantage of Time Decay

For example, a *calendar spread* takes advantage of this: sell the near term call and buy the far term call at the same strike. The trade is for a debit because the far term call is more expensive.

But the near term call's time value decays faster; and if ATM, it decays to zero.

Factors —Volatility

Volatility is a measure of how rapidly and how widely a stock price swings up and down. The bigger and more frequent the swings, the greater the chance the option will be ITM, at least at some point. Hence high volatility increases the price of an option, or any combination of options.

Historical vs Implied Volatility

The volatility that should be used in figuring an option's price initially is its *historical volatility*. This is a calculation of the price swings of the underlying in the recent past.

But then the market takes over and controls the price literally from moment to moment. What becomes of the volatility then?

If the volatility determines the price, then one can go the other way around and use the price to figure the volatility. This is called the *implied volatility*.

Implied Volatility is the Instantaneous Volatility

By way of analogy, to compute the speed of a car one divides how far it moves over a short span of time by that period of time. This gives the average speed over the preceding time span. But by looking at the car's speedometer one gets the instantaneous speed right now.

In the same way historical volatility is the average volatility over a preceeding time span, while implied volatility is the instantaneous volatility of the underlying right now (as deemed by the market).

One learns to get a sense as to whether the option is priced high or low by its implied volatility.

Factors — Risk Free Rate

It might be surprising that the current T-bill rate should have an effect on option prices, but it does. This is because options compete with other investments, T-bills among them.

(Question: why is it that the T-bill rate rather than the stock's own upward drift that affects the price?)

The T-bill rate serves as the so called *risk free rate* (RFR).

As the RFR increases, so does the option price. But the RFR has only a weak influence on prices as seen in the following table.

Parameters: stock price 100.00, strike price 100.00, expiry 60 days, volatility 40

riskfree rate	call price	ITM prob.
0.020	6.62	0.476
0.024	6.65	0.477
0.028	6.68	0.479
0.032	6.71	0.481
0.036	6.74	0.482
0.040	6.77	0.484
0.044	6.81	0.485
0.048	6.84	0.487
0.052	6.87	0.489
0.056	6.90	0.490
0.060	6.93	0.492

Mathematical Factors — the Natural Logarithm

The Black-Scholes calculation of an option price makes use of the natural logarithm – why is that?

This occurs for the same reason that the natural logarithm is used to compute compound interest, namely because price changes in the underlying are proportional to the price of the underlying.

Mathematical Factors — the Normal Distribution

The Black-Scholes calculation of an option price makes use of the *normal probability distribution*. This is because in the Black-Scholes model, price movements are assumed to be normally distributed — small movement with high probability and large movement with very low probability.

But what about the Black Swan effect?

This is not accounted for by the Black-Scholes model and may be a deficiency in the model. If taken into account, it would essentially raise the volatility and increase option prices.

Impoderables

- the news
- crowd psychology
- other . . .